

Bruce Wei

P42-69

$$y = Ce^{kt}$$

$$a) \begin{cases} 125 = Ce^{2k} \\ 350 = Ce^{4k} \end{cases}$$
$$e^{2k} = \frac{350}{125} = \frac{14}{5}$$

$$k = \frac{1}{2} \ln \frac{14}{5} \approx 0.515$$

$$C = \frac{y}{e^{0.515t}}$$
$$= \frac{125}{e^{0.515 \times 2}}$$

$$\hat{=} 45$$

$$b) y = 45e^{0.515t}$$

$$c) y = 45 \cdot e^{0.515 \times 8}$$
$$= 2770$$

$$d) 45e^{0.515t} = 25000$$

$$t = 12.27$$

Exercises 33–40, complete the table for

Initial Quantity	Amount After 1000 Years	Amount After 10,000 Years
20 g	1.5 g	0.1 g
5 g	1.6 g	3 g
	2.1 g	0.4 g

Radioactive radium has a half-life of 1580 years. What percent of a given amount remains after 1000 years?

Carbon-14 dating assumes that the carbon in an ancient object has the same radioactive content as it has today. If this is true, the amount of ^{14}C absorbed by a tree several hundred years ago should be the same as the amount absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much of the radioactive isotope as a piece of modern charcoal. How long ago was the charcoal made? (The half-life of ^{14}C is 5730 years.)

Exercises 43–48, complete the table for which interest is compounded continuously.

Annual Percentage Rate	Time to Double	Amount After 10 Years
7%	$7\frac{3}{4}$ yr	\$1292.85
8%	5 yr	\$5436.56

Exercises 49–52, find the principal P that must be invested at rate r , compounded monthly, so that it will amount to A dollars after t years.

- 50. $r = 6\%$, $t = 40$
- 52. $r = 9\%$, $t = 25$

In Exercises 53–56, find the time necessary for an investment of P dollars to grow to A dollars if it is invested at a rate of r compounded (a) annually, (b) quarterly, (c) daily, and (d) continuously.

- 54. $r = 6\%$
- 56. $r = 5.5\%$

Population In Exercises 57–61, the population (in millions) of a country in 2007 and the expected continuous annual rate of change k of the population are given. (Source: U.S. Census Bureau, International Data Base)

- (a) Find the exponential growth model $P = Ce^{kt}$ for the population by letting $t = 0$ correspond to 2000.
- (b) Use the model to predict the population of the country in 2015.
- (c) Discuss the relationship between the sign of k and the change in population for the country.

Country	2007 Population	k
57. Latvia	2.3	-0.006
58. Egypt	80.3	0.017
59. Paraguay	6.7	0.024
60. Hungary	10.0	-0.003
61. Uganda	30.3	0.036

CARSTONE

- 62. (a) Suppose an insect population increases by a constant number each month. Explain why the number of insects can be represented by a linear function.
- (b) Suppose an insect population increases by a constant percentage each month. Explain why the number of insects can be represented by an exponential function.

63. **Modeling Data** One hundred bacteria are started in a culture and the number N of bacteria is counted each hour for 5 hours. The results are shown in the table, where t is the time in hours.

t	0	1	2	3	4	5
N	100	126	151	198	243	297

- (a) Use the regression capabilities of a graphing utility to find an exponential model for the data.
- (b) Use the model to estimate the time required for the population to quadruple in size.

64. **Bacteria Growth** The number of bacteria in a culture is increasing according to the law of exponential growth. There are 125 bacteria in the culture after 2 hours and 350 bacteria after 4 hours.

- (a) Find the initial population.
- (b) Write an exponential growth model for the bacteria population. Let t represent time in hours.
- (c) Use the model to determine the number of bacteria after 8 hours.
- (d) After how many hours will the bacteria count be 25,000?

65. **Learning Curve** The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is $N = 30(1 - e^{-kt})$. After 20 days on the job, a particular worker produces 19 units.